Comparison of Inviscid and Viscous Separated Flows

J. A. Rhodes*
McDonnell Aircraft Company, St. Louis, Missouri
and
E. von Lavante†
Old Dominion University, Norfolk, Virginia

Abstract

INVISCID compressible flow separation on simple two-dimensional configurations has been investigated by solving the Euler equations numerically, using different computational schemes. The results were compared with numerical solutions of the Navier-Stokes equations. The computational schemes used in this study included a lambda scheme with shock fitting, a flux-vector splitting scheme for the Euler equations, and a flux-vector splitting scheme for the Navier-Stokes equations. The results show significant difference between the viscous and inviscid flows, indicating different phenomenological causes.

Contents

The present investigation focuses on the so-called "inviscid separation" phenomenon as observed on several two-dimensional geometries such as backward facing step, circular arc airfoil, and channel flow. It attempts to provide answers to the questions of uniqueness and validity as well as physical relevance of solutions of the compressible Euler equations that show flow separation. Only the backward facing step case will be discussed here; more details of the other test cases are presented in the full version of this paper.[1] The physical geometry and grid were obtained using a conformal transformation given by

$$z = \{(\zeta^2-1)^{1/2} + \ln[\zeta + (\zeta^2-1)^{1/2}]\}/\pi$$

Lines of constant ξ and η were used to define the physical and corresponding computational domain. The minimum value of η (η_0) determines how close the lower boundary gets to a step with a sharp corner that occurs at $\eta_0 = 0$. The lower boundary was kept above this value for all runs in order to provide a smoother grid at the corner and produces a more gradual expansion. The upper value of $\eta(\eta_1)$ determined where the upper boundary was, and computations were carried out at various values of η_1. The grid-point distribution along the boundaries was determined by polynomial and exponential stretchings.

Three schemes were used to solve the Euler equations. Scheme 1 is an explicit lambda scheme of Moretti[2] and uses the three-step predictor-corrector procedure of Gabutti.[3] The flow is assumed isenthalpic and the equations are solved in an upwind but nonconservative form. Scheme 2 is a shock-fitting version of scheme 1. An equation for the conservation of entropy along streamlines in shock-free regions is added in place of the isenthalpic assumption. The shock fitting is done on a grid that adapts to the moving embedded shock and the shock acceleration, and velocity is calculated at each step. Scheme 3 is an implicit finite-volume flux vector-splitting algorithm that solves the Euler equations in conservation form. The form is assumed isenthalpic, and van Leer splitting with MUSCLE differencing is used.[4]

The viscous calculations were done using an implicit, upwind finite-volume scheme developed by Rumsey.[5] The code solves the full Navier-Stokes equations, and the flux splitting of van Leer is used again for the inviscid part. The viscous terms are centrally differenced. The code has been shown to give good results for steady and unsteady flows.

The upper and lower boundaries were treated as inviscid walls by the three Euler schemes. The viscous code treated the lower and upper boundries as no-slip and freestream, boundaries respectively. The pressure was specified at the downstream boundary by all of the codes. The three Euler schemes, however, combined this with a nonreflecting condition in which the pressure was allowed to fluctuate about the specified value in order to allow waves to pass through the outflow boundary. A nondimensional static pressure p/p_o equal to 0.7 was used in all cases.

Along the upstream inflow boundary, all codes assumed the v component of velocity to be zero and the flow to be isenthalpic. Schemes 1 and 2 used the steady one-dimensional energy equation to relate the change in pressure to the change in the u component of velocity. The flux-vector splitting scheme used the concept of Riemann invariants at the inflow boundary. The u component was simply specified in the Navier-Stokes code.

Results and Discussion

The three inviscid codes were run using various values of η_0 at the lower boundary. By lowering η_0, the expansion corner becomes sharper and more prone to separation. The effect of the position of the upper wall on the corner flow was determined by changing the extent of the physical domain in the upward direction. It was determined through numerical experimentation that as the upper boundary (η_1) was increased from 20 to 100, the influence of this boundary was lessened. This is because the area ratio of the inflow to the outflow boundary asymptotes to 1 as the upper wall is increased. The results obtained, however, suggest that the influence of the upper boundary on the nature of the flow solution at the corner was relatively weak, assuming that the distance between the two walls is sufficiently large. An 81×41 grid was used for most of the calculations; scheme 2 was also run on a refined grid with 161×81 lines. The results on the fine grids were not substantially different from the coarser grids.

At the largest value of $\eta_0 = 2.0$, the corner is very smooth and the flow predicted by all three Euler solvers remains attached. A shock forms after the apex of the step; its strength is nearly identical for all schemes.

As η_0 is decreased, the corner becomes sharper, eventually leading to separation. Scheme 2 first separated at $\eta_0 = 1.52$; the flux-vector splitting scheme (3) separated at $\eta_0 = 1.12$. The lower value of η_0 needed for separation with scheme 3 is due to the effects of the inherent artificial dissipation on the entropy gradients along the shock. The Mach number contours showing the resulting flowfield as predicted by the scheme 2 for $\eta_0 = 1.2$ can be seen in Fig. 1. Clearly visible are the fitted

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*Research Engineer, MEM Dept. Member AIAA.
†Associate Professor, MEM Dept. Member AIAA.

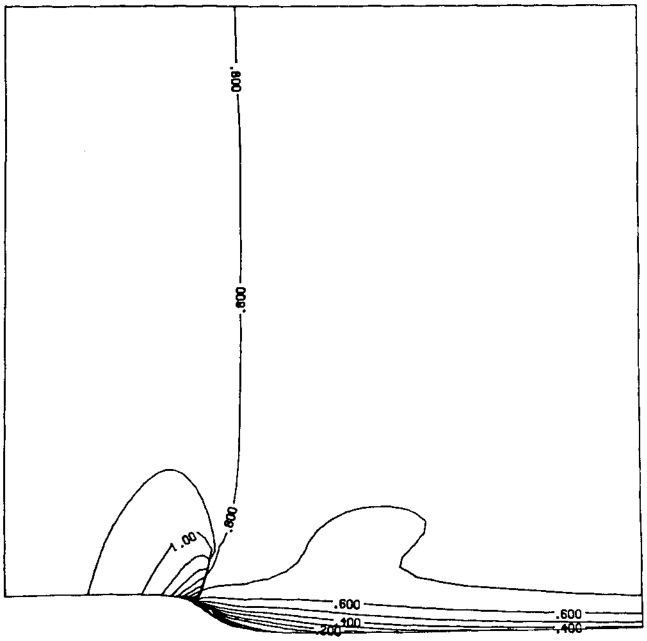

Fig. 1 Mach number contours of results from scheme 2 (shock-fitting), $\eta_0 = 1.2$, $\eta_1 = 40$, showing "inviscid separation."

shocks and the region of separated flow. This separation bubble is relatively small, since incipient separation occurred at $\eta_0 = 1.52$.

Further decrease of η_0 results in an increase of the size of the separation bubble as the reattachment point moves downstream. The shock strength increases when both the shock-capturing and shock-fitting schemes are used until an η_0 of 1.2 is reached. After this point, the shock strength continues to increase with shock-fitting scheme 2. In the case of scheme 3, the strength gradually decreases and approaches in the limit η_0 zero strength, and the shock location goes to $\xi = 0$, which is the tip of the corner. In this case, the recirculating region "fills" out the step such that the streamlines leave the corner smoothly and the flow is shockless.

In all separated cases, the steady-state results show a closed recirculation region and a flow region external to it, with no apparent momentum exchange between them except for the effects of artificial damping. The vorticity downstream of the shock is directly proportional to the entropy gradient along the shock. In the case of scheme 2, the shock is strongly curved at the wall, producing a layer of high-entropy gradient. It is assumed that this leads to transient development of the separated zone that is eventually trapped inside the closed eddy after reaching steady state.

The results obtained from the viscous code in a laminar mode for a Reynolds number of 10^4 are shown in Fig. 2. The upper and lower boundries are in the same locations as in the inviscid case, and only the upstream boundary has been extended to allow the boundary layer to develop. It can be seen from the Mach number contours in Fig. 2 that the flow has separated, and there is no shock at the expansion corner. Furthermore, an examination of the solution at increments of 500 iterations leads the authors to conclude that the flow is unsteady and periodic in nature with vorticities forming and being shed from the corner.

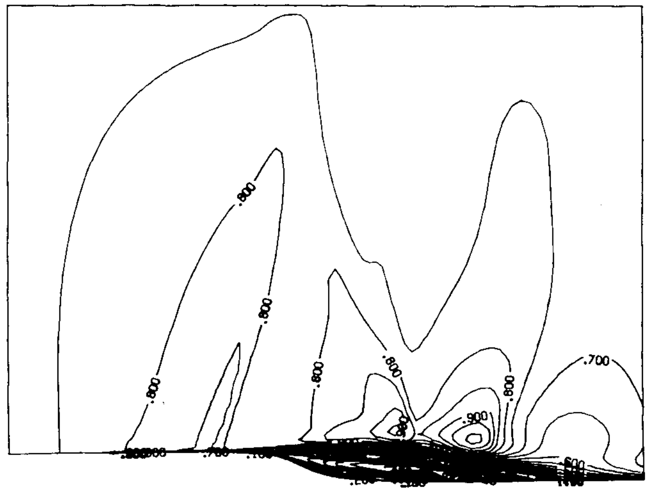

Fig. 2 Mach number contours for viscous results, $\eta_0 = 2.0$, $\eta_1 = 40$, $Re = 10^4$.

Conclusions

The results lead the authors to conclude that the inviscid flow separation phenomenon should not be viewed as a prescursor or a predictor to viscous separation. The fact that the two different computational schemes for solving the Euler equations showed different behavior indicates that the inviscid separation is at least partially of numerical origin; however, the results also suggest that the correlation between the entropy gradient along the shock and vorticity downstream of the shock is indicative of physical mechanism for flow separation. This has been shown to exist by the work of Frankel.[6]

In the present cases, the laminar viscous flow as predicted by the Navier-Stokes solver was significantly different from the Euler results. The viscous separation was based on different physical phenomena, and any inviscid separation must be carefully interpreted.

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